A Lean Formalization of Cedar

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CEDAR

Cedar is an open source authorization policy language, developed at Amazon Web Services (AWS). Cedar allows for controlled access to resources via a simple and expressive syntax that supports different authorization paradigms, such as attribute-based access control (ABAC) and role-based access control (RBAC). Cedar policies define who (the **principal**) can do what (the **action**) on what target (the **resource**) when (the context). Policies have a specified effect: either **permit** or **forbid**.

```
permit(principal in Group::"admins",
action in [Action::"create", Action::"delete"],
resource in Pages::"admin_pages")
unless(principal in Group::"blacklisted_admins");
```

This example policy *permits* a principal with the `admins` role to perform a `create` or `delete` action on resources that have the `admin_pages` role, unless the principal also has the `blacklisted_admins` role.
Cedar allows a user to define such a set of policies, and then make an authorization request. The request is either **allowed** or **denied**, based on a set of rules. Informally, an authorization request could be of the form "Is `Alex` allowed to create the admin page `Instructions`?", which will be allowed if and only if `Alex` is in the group `admins` and not in the group `blacklisted_admins`.

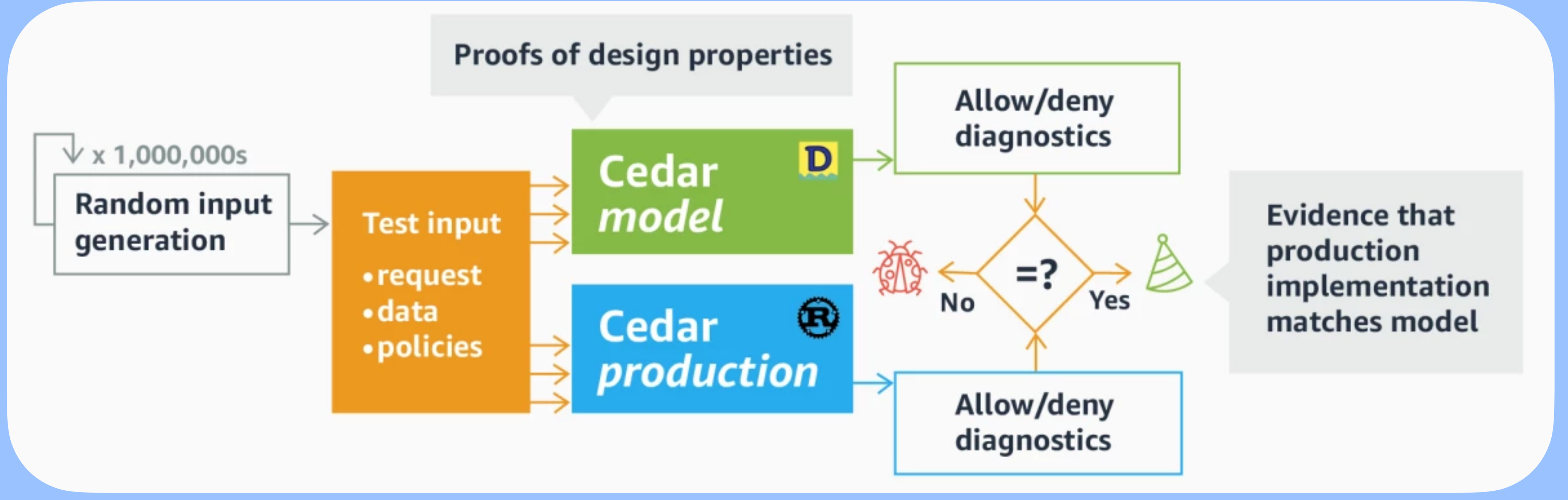
Cedar is dynamically typed, and type-checking takes place during an optionally run phase called validation.

Dafny is a verification-aware programming language. Dafny makes use of automated reasoning, allowing programmers to reason about their code formally by making use of specifications. Dafny discharges proof obligations to an SMT solver, Z3, allowing additional pre and post conditions and assertions to assist the solver.

Lean is a proof assistant and functional programming language. Lean is an interactive theorem prover, allowing users to write proofs via direct construction or tactics, which are then checked by the Lean kernel. Lean's type theory is based on the calculus of inductive constructions.

VERIFICATION GUIDED DEVELOPMENT

Cedar uses a process called *verification-guided development*, to ensure the correctness of the authorization engine. The authorizer and validator are modeled in Dafny, and using Dafny's automated reasoning capabilities, a collection of security properties are checked and proved. Via *differential random testing (DRT)*, the production implementation in Rust is checked for equivalence with the Dafny model. In Cedar, 25 bugs have been found through DRT, and 4 bugs through failed proof attempts.



AN ALTERNATIVE VERIFICATION ENVIRONMENT

Dafny was chosen for its balance between usability and automation for basic properties. However, meta-theoretic properties of Cedar have proved less suitable for Dafny's automation. The specification suffered from poor proof performance and brittleness, where small changes to the program, or minor updates to Dafny or Z3 caused verification timeouts. Proof brittleness is a well known issue with SMT-based tools such as Dafny. To ensure robust performance and minimal maintenance, highly detailed proofs are better, and this favors the use of an interactive theorem prover over an automated one. We port the Cedar formalization to the interactive theorem prover Lean, and try to answer the question: **Can Lean be used for verifying a project at the scale of Cedar, with performance and proof size metrics comparable to the existing formalization in Dafny?**

LANGUAGE SPECIFICATION

The language specification is made up of the authorization and evaluation models. The authorization model is quite simple: we collect the list of forbid and permit policies satisfied; if there is at least one permit policy satisfied, and no forbid policies are satisfied, then the authorizer allows the request, else it denies it.

```
def isAuthorized (req : Request) (entities : Entities) (policies : Policies) :
Response :=
let forbids := satisfiedPolicies .forbid policies req entities
let permits := satisfiedPolicies .permit policies req entities
if forbids.isEmpty && !permits.isEmpty
then { decision := .allow, policies := permits }
else { decision := .deny, policies := forbids }
```

A request is evaluated against each policy in the given policy set. Evaluation can return either true, false, or error. Each constraint in the policy scope is an expression; members of the context also form expressions. Unconstrained principal, action, or resource clauses evaluate to true.

We prove the following theorems for the authorizer:

- If some forbid policy is satisfied, then the request is denied.
- A request is allowed if and only if it is explicitly permitted (i.e., there is at least one permit policy that is satisfied).
- Authorization produces the same result regardless of policy evaluation order or duplicates.

VALIDATION MODEL

In the existing Dafny formalization, Cedar validation followed a slightly complex model known as *permissive validation*. During the Lean formalization, we decided to change the validation model to be *stricter*, by simplifying the type system quite a bit. The only non-standard part of the type system is to do with booleans: when we are able to make certain judgements, we type boolean values *strongly* with `tt` and `ff` representing the **true** and **false types**, over and above the regular anyBool type which corresponds to the more familiar boolean type.

```
inductive BoolType where
| anyBool
| tt
| ff
```

The subtyping relation is also simple. Records have width subtyping, but not depth subtyping; `tt <: anyBool` & `ff <: anyBool`; every type is a subtype of itself.

STATISTICS

MODEL	DAFNY LOC	LEAN LOC	%
Generic datatype definitions	0	246	
Language model specification	1707	951	56%
Validation model specification	1189	532	45%
Total	2896	1729	60%

PROOFS	DAFNY LOC	LEAN LOC	%
Datatype proofs	0	681	
Authorizer proofs	394	350	89%
Validator proofs	3110	4686	150%
Total	2896	1729	160%

VERIFICATION TIME	DAFNY (S)	LEAN (S)	%
	519	185	36%

CODE

EXTENDED ABSTRACT

DRT: PER REQUEST	LEAN (µs)	DAFNY JAVA (µs)
abac	4	3325
abac-typed	5	3410

The Lean specification outperforms the Dafny specification – this can be attributed to the use of type classes and higher order functions, as well as Lean's extensive standard library. Dafny proofs are mostly shorter than Lean proofs – this was expected, and is attributed to Dafny's ability to automatically solve simple proof obligations via an SMT solver. Both verification time and time per test request for Lean were significantly lower than Dafny, which can be attributed to the difference in underlying compilers. Overall, the Lean specification performs extremely well in all regards and is a significant improvement especially with respect to the differential random testing that Cedar relies on, as more tests can be run in the daily fixed period.

AN ASIDE: MODELING SETS

Cedar makes heavy use of sets, and in Dafny, sets are axiomatized; in Lean, this is not the case. We have a type `Value` where `set : Set Value -> Value`, that is, we have a constructor that takes in a set of values as a parameter. To define a quotient type representing sets on Value, we would need to define a function and a type in a mutually recursive fashion, something that is not permitted in Lean. Hence, we had to settle on an alternative definition: a set is a wrapper around a list, but we only deal with well-formed sets, that is, sets where the underlying list is sorted and duplicate free.

```
inductive Value where
...
| set (s : Set Value)
...
```