Imperative Syntax for Dependent Types

Towards a user study...

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Dependent Types

a.k.a. types that can depend on terms

```
data List : Type -> Type where
  Nil : List a
  Cons : a -> List a -> List a
```

↓

```
data Vect : Nat -> Type -> Type where
  Nil : Vect 0 a
  Cons : a -> Vect n a -> Vect (S n) a
```

“Dependent types are HARD”

“Dependent types are HARD”

But WHY?

“Dependent types are HARD”

But WHY?

And for WHOM?

"Dependent types are HARD"

But WHY?

And for WHOM?

It depends.

An *Experienced** Imperative Programmer

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a.k.a our desired audience

- Is able to comprehend complex imperative programming features

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```
#include<iostream>
using namespace std;

template <long N> struct Factorial
{
    enum { value = N * Factorial<N - 1>::value };
};

template <> struct Factorial<0>
{
    enum { value = 1 };
};

int main()
{
    cout << Factorial<15>::value << endl;
    return 0;
}
```

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- Ideally, cares a bit about correctness

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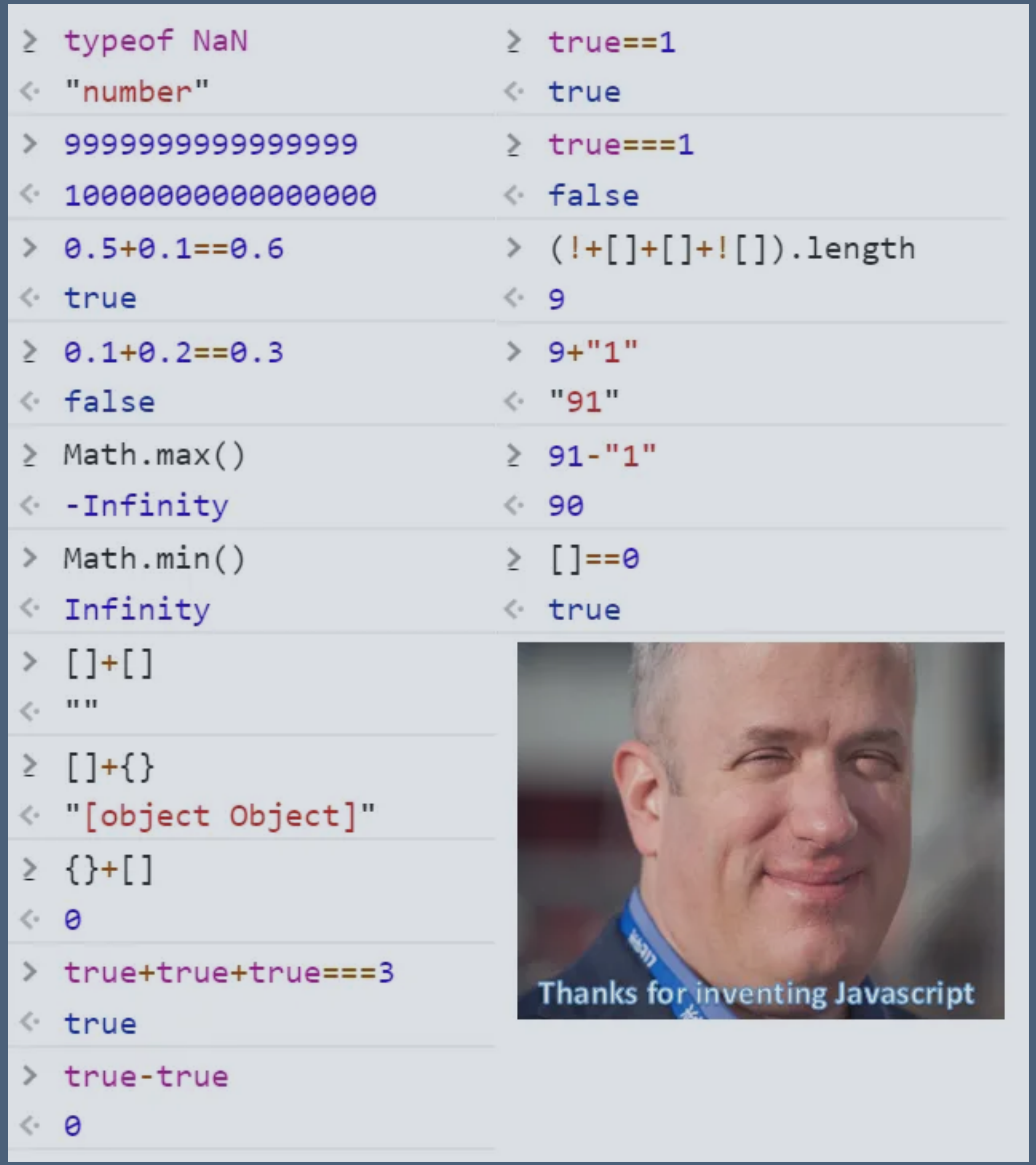
Not this guy ->



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```
// Callback Hell


a(function (resultsFromA) {
    b(resultsFromA, function (resultsFromB) {
        c(resultsFromB, function (resultsFromC) {
            d(resultsFromC, function (resultsFromD) {
                e(resultsFromD, function (resultsFromE) {
                    f(resultsFromE, function (resultsFromF) {
                        console.log(resultsFromF);
                    })
                })
            })
        })
    })
});
```

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- ??????

Before dependent types

a.k.a functional programming

- Most dependently typed languages use typically functional syntax
- Imperative and functional languages differ significantly in style
- While implementation details may differ, many core concepts exist within both paradigms

Before dependent types

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- Most dependently typed languages use typically functional syntax
- Imperative and functional languages differ significantly in style
- While implementation details may differ, many core concepts exist within both paradigms
- An (important) example: Algebraic data types and pattern matching

Algebraic Data Types

In imperative languages

```
@dataclass
class Nil:
  pass
@dataclass
class Cons:
  head: int
  tail: List
List = Nil | Cons
```

Python: *dataclasses & tagged unions*

Algebraic Data Types

In imperative languages

```python
@dataclass
class Nil:
  pass
@dataclass
class Cons:
  head: int
  tail: List
List = Nil | Cons
```

Python: *dataclasses & tagged unions*

```cpp
struct Nil final {};
struct Cons final {
  int head;
  std::unique_ptr<std::variant<Nil, Cons>> tail;
};
using List = std::variant<Nil, Cons>;
```

C++: *structs & variants*

Algebraic Data Types

In imperative languages

```cpp
struct Nil final {};
struct Cons final {
  int head;
  std::unique_ptr<std::variant<Nil, Cons>> tail;
};
using List = std::variant<Nil, Cons>;
```

C++: *structs & variants*

```python
@dataclass
class Nil:
  pass
@dataclass
class Cons:
  head: int
  tail: List
List = Nil | Cons
```

Python: *dataclasses & tagged unions*

```java
sealed interface List<T> {
  record Nil<T>() implements List<T> {}
  record Cons<T>(T head, List<T> tail)
implements List<T> {}
```

Java: *sealed interfaces & records*

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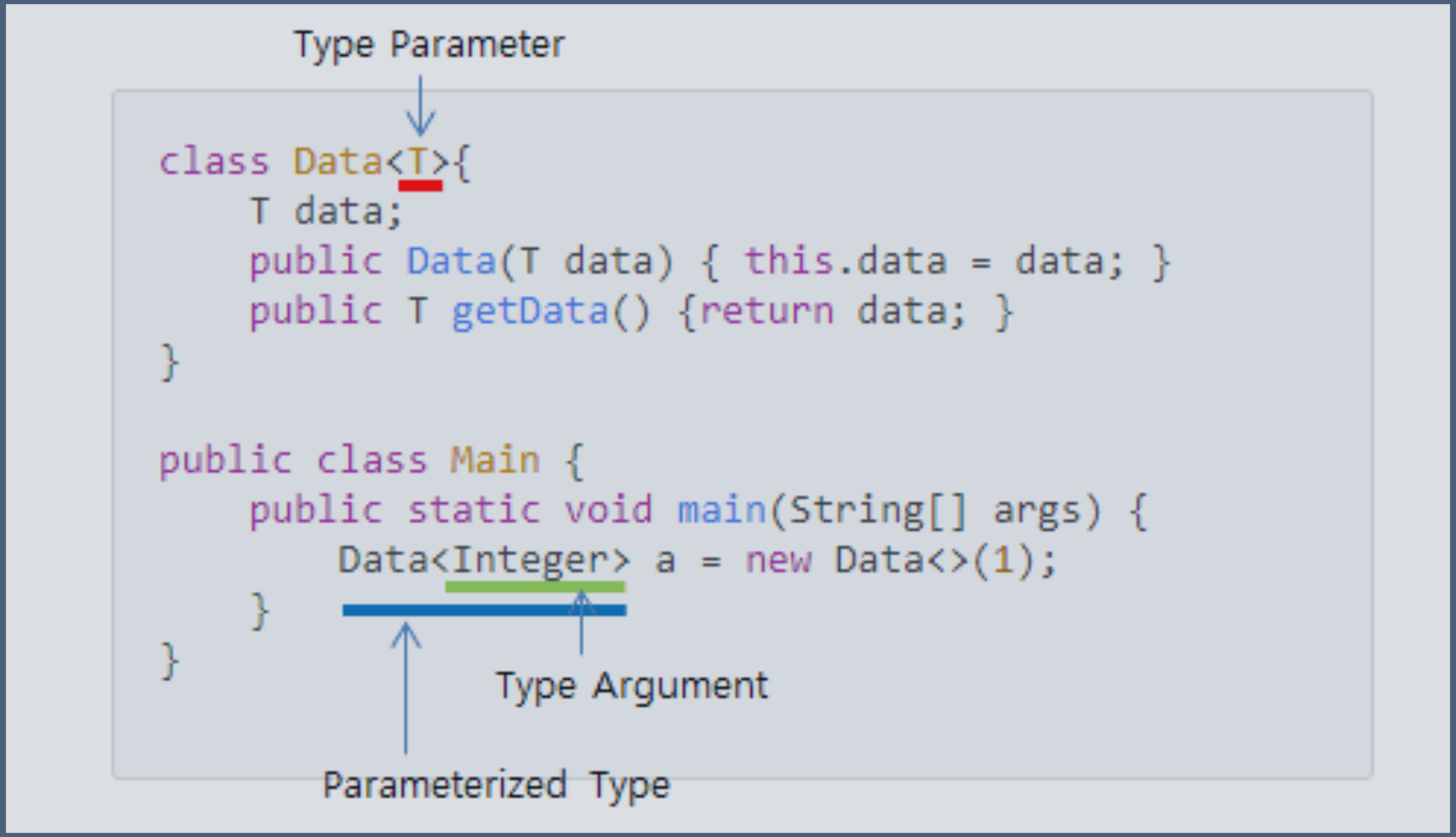
- ??????



Imperative syntax for an existing dependent type theory can enhance usability for an experienced programmer.

The Experiment

- A primarily syntactic imperative layer that is elaborated into an existing dependently typed language (Idris)
- To more rigorously define an equivalence between the imperative layer and existing Idris code
- Without changing the underlying dependent type theory
- Allowing for a (read-only) comparison of syntax without changing the semantics
- Via a principled empirical user study of experienced imperative programmers

Syntax

Simple syntactic transformations

a.k.a. the low hanging fruit

```
type Vect(Nat n, Ty t) {
  constructor Nil() of Vect(0, t);
  constructor Cons(t head, Vect(n, t) tail) of Vect(n+1, t);
}
```

↓ *Datatypes*

```
data Vect : Nat -> Type -> Type where
  Nil : Vect 0 t
  Cons : (head : t) -> (tail : Vect n t) -> Vect (S n) t
```

Simple syntactic transformations

a.k.a. the low hanging fruit

```
func replicate<Ty t>(t x, Nat n) of Vect(t, n) {
  switch(n) {
     case 0: { return Nil; }
     case S(n): { return Cons(x, replicate(x,n)); }
  }}
```

↓ *Pattern matching*

```
replicate : {t : Type} -> t -> Nat -> Vect t n
replicate x n = case n of
   0 => Nil
   S n => Cons x (replicate x n)
```

Simple syntactic transformations

a.k.a. the low hanging fruit

```
func varManip (Nat x, Nat y) of Nat {
  let Nat z = x + y;
  if (z < 10) {
      z = z + 10;
  } else {
      z = z + 1;
  }
  z = z + x;
  return z;
}
```

Block statements / variable reassignment →

```
varManip : Nat -> Nat -> Nat
varManip x y =
 let z : Nat = (x + y) in
  if (z < 10) then
   let z : Nat = (z + 10) in
    let z : Nat = (z + x) in
     z
  else
   let z : Nat = (z + 1) in
    let z : Nat = (z + x) in
     z
```

Loops

a.k.a recursion

```
func f(t1 x, t2 y, …) of t {
  head
  while(condition) {
    body
  }
  tail
}
```

Loops

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    body
  }
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}
```

```
func f(t1 x, t2 y, …) of t {
  head
  f'(x, y, …, xh, yh, …)
}

func f'(t1 x, t2 y, …, th1 xh, th2 yh, …) of t {
  if(condition) {
    body
    f'(x, y, …, xh, yh, …)
  } else {
    tail
  }
}
```

Loops

a.k.a recursion

```
func f(t1 x, t2 y, …) of t {
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func f'(t1 x, t2 y, …, th1 xh, th2 yh, …) of t {
  if(condition) {
    body
    f'(x, y, …, xh, yh, …)
  } else {
    tail
  }
}
```

From booleans to propositions

a.k.a enabling dependent pattern matching

```
if(x == Nil) {
  return 0;
} else {
  return head(x);
}
```

From booleans to propositions

a.k.a enabling dependent pattern matching

```
eif(x == Nil) {
  return 0;
} else {
  return head(x);
}
```

From booleans to propositions

a.k.a enabling dependent pattern matching

```
eif(x == Nil) {
  return 0;
} else {
  return head(x);
}
```

→

```
case (decEq x Nil) of
  Yes prf => 0
  No  prf => head x
```

```
head : (x : List Nat) -> {_ : Not (x = Nil)} -> Nat
```

Decidable Equality...

```
data List : (t : Type) -> Type where
    Nil : List t
    Cons : (head : t) -> (tail : List t) -> List t
```

Decidable Equality…

```
data List : (t : Type) -> Type where
    Nil : List t
    Cons : (head : t) -> (tail : List t) -> List t

(DecEq t) => DecEq (List t) where
    decEq Nil Nil = Yes Refl
    decEq (Cons h1 t1) (Cons h2 t2) with (decEq h1 h2)
        decEq (Cons h1 t1) (Cons h1 t2) | Yes Refl with (decEq t1 t2)
            decEq (Cons h1 t1) (Cons h1 t1) | Yes Refl | Yes Refl = Yes Refl
            decEq (Cons h1 t1) (Cons h1 t2) | Yes Refl | No prf = No $ (\h => prf
(case h of Refl => Refl))
        decEq (Cons h1 t1) (Cons h2 t2) | No prf = No $ (\h => prf (case h of Refl
=> Refl))
    decEq Nil (Cons h t) = No (\h => (case h of Refl impossible ))
    decEq (Cons h t) Nil = No (\h => (case h of Refl impossible ))
```

Decidable Equality…

```
data SoManyArgs : (t : Type) -> Type where
  C1 : (a : t) -> (b : t) -> (c : t) -> (d : t) -> SoManyArgs t
  C2 : (x : t) -> (y : t) -> SoManyArgs t

{t : Type} -> (DecEq t) => DecEq (SoManyArgs t) where
  decEq (C1 a1 b1 c1 d1) (C1 a2 b2 c2 d2) with (decEq a1 a2)
    decEq (C1 a1 b1 c1 d1) (C1 a1 b2 c2 d2) | Yes Refl  with (decEq b1 b2)
      decEq (C1 a1 b1 c1 d1) (C1 a1 b1 c2 d2) | Yes Refl | Yes Refl  with (decEq c1 c2)
        decEq (C1 a1 b1 c1 d1) (C1 a1 b1 c1 d2) | Yes Refl | Yes Refl | Yes Refl  with (decEq d1 d2)
          decEq (C1 a1 b1 c1 d1) (C1 a1 b1 c1 d1) | Yes Refl | Yes Refl | Yes Refl | Yes Refl  = Yes Refl
          decEq (C1 a1 b1 c1 d1) (C1 a1 b1 c1 d2) | Yes Refl | Yes Refl | Yes Refl | No prf  = (No (\h => (prf (case h of Refl
=> Refl))))
        decEq (C1 a1 b1 c1 d1) (C1 a1 b1 c2 d2) | Yes Refl | Yes Refl | No prf  = (No (\h => (prf (case h of Refl => Refl))))
      decEq (C1 a1 b1 c1 d1) (C1 a1 b2 c2 d2) | Yes Refl | No prf  = (No (\h => (prf (case h of Refl => Refl))))
    decEq (C1 a1 b1 c1 d1) (C1 a2 b2 c2 d2) | No prf  = (No (\h => (prf (case h of Refl => Refl))))
  decEq (C1 a1 b1 c1 d1) (C2 x2 y2) = (No (\h => (case h of Refl impossible)))
  decEq (C2 x1 y1) (C1 a2 b2 c2 d2) = (No (\h => (case h of Refl impossible)))
  decEq (C2 x1 y1) (C2 x2 y2) with (decEq x1 x2)
    decEq (C2 x1 y1) (C2 x1 y2) | Yes Refl  with (decEq y1 y2)
      decEq (C2 x1 y1) (C2 x1 y1) | Yes Refl | Yes Refl  = Yes Refl
      decEq (C2 x1 y1) (C2 x1 y2) | Yes Refl | No prf  = (No (\h => (prf (case h of Refl => Refl))))
    decEq (C2 x1 y1) (C2 x2 y2) | No prf  = (No (\h => (prf (case h of Refl => Refl))))
```

Decidable Equality for FREE

```
type Vect(Nat n, Ty t) {
  constructor Nil() of Vect(0, t);
  constructor Cons(t head, Vect(n, t) tail) of Vect(n+1, t);
}
```

Datatypes with DecEq declarations :)

```
data Vect : Nat -> Type -> Type where
  Nil : Vect 0 t
  Cons : (head : t) -> (tail : Vect n t) -> Vect (S n) t

(DecEq t) => DecEq (Vect t) where
    …
```

Putting it all together

```
func search (Nat n, Vect(n, Nat) ls, Nat x) of Maybe(Fin(n)) {
  let Nat i = 0;
  let Maybe(Fin(n)) ret = Nothing;
  ewhile(i < n) {
    eif (index(natToFinLT(i), ls) == x) {
      ret = Just(natToFinLT(i));
    }
    else { ;; }
    i = 1 + i;
  }
  return ret;
}
```

Putting it all together

```
search : (n : Nat) -> (ls : Vect n Nat) -> (x : Nat) -> Maybe (Fin n)
search n ls x =
  let i : Nat = 0 in
    let ret : Maybe (Fin n) = Nothing in
      (search_rec0 n ls x i ret)
where
  search_rec0 : (n : Nat) -> (ls : Vect n Nat) -> (x : Nat) -> (i : Nat)  -> (ret : Maybe (Fin
n)) -> Maybe (Fin n)
  search_rec0 n ls x i ret =
    (case (isLT i n) of
      No noprf => ret
      Yes yesprf => (case (decEq (index (natToFinLT i) ls) x) of
        No noprf => let i : Nat = (S i) in
          (search_rec0 n ls x i ret)
        Yes yesprf => let ret : Maybe (Fin n) = Just (natToFinLT i) in
          let i : Nat = S i in
            search_rec0 n ls x i ret))
```

Syntax isn't everything

a.k.a. type theory matters

- Errors!!!!
- Semantics of effects (eg. mutability)
- Interactive type-checking (explicit proofs)
- And more …

A study

What does it look like?

- Target participants: experienced imperative programmers
  - Choice of (imperative) language with maximum experience
  - Primary context of programming experience
- A purely syntactic comparison
  - Evaluation of usability without interactivity
  - Imperative-style programs in functional languages (and vice versa)

Conclusion

- Designed an imperative syntax for dependently typed programming that can be elaborated to executable Idris code
- Developed an algorithm for automatic derivation of decidable equality
- Syntax isn't everything — dedicated semantics are necessary for true usability
  - Specifically, we need a better understanding of *how* imperative programmers would make use of dependent types
- This syntax is to be evaluated via a qualitative user study